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FIZIK CHIZIQLI QOVUSHOQ ELASTIK STERJENNING AVTOTEBRANISH MASALASINING MATEMATIK MODELI

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Annotation. This article considers the problem of modeling self-oscillating processes of a physical linear viscoelastic rod in a gas flow, taking into account linear dependencies.
Keywords: viscoelasticity, rod, self-oscillations, physical linearity, aerodynamic linearity, Bubnov-Galerkin method, relaxation kernel, numerical method, critical velocity.
Аннотация. В данной статье рассматривается задача моделирования автоколебательных процессов физического линейного вязкоупругого стержня в потоке газа с учетом линейных зависимостей.
Ключевые слова: вязкоупругость, стержень, автоколебания, физическая линейность, аэродинамическая линейность, метод Бубнова-Галёркина, ядро релаксации, численный метод, критическая скорость.
Annotatsiya. Ushbu maqolada chiziqli bog'liklarni hisobga olgan holda gaz oqimidagi fizik chiziqli qovushqoq elastik sterjenning avtotebranish jarayonlarining modeli haqidagi masala ko'rilmoqda.
Tayanch so'zlar: qovushqoq-elastik, sterjen, avtotebranish, fizik chiziqlilik, aerodinamik chiziqlilik, Bubnov-Galyorkin usuli, relaksatsiya yadrosi, sonli usul, kritik tezlik.

Kirish. Qovushqoq-elastikning irsiy nazariyasi turli materiallarning deformatsiyalanishining dinamik jarayonlarini tavsiflash uchun keng imkoniyat yaratdi. Sterjenlar sanoat va texnologiyaning ko'plab sohalarida konstruktiv elementlar sifatida qo'llanilganligi sababli, ularning turli shakllardagi dinamik harakatlarini o'rganish va materialning fizik chiziqlilik xususiyatini hisobga olgan holda avtotebranishlar va dinamik barqarorlik uchun konstruksiyalarni o'rganish dolzarbdir.

Masalaning qo'yilishi. Fizik chiziqlilik xususiyatini hisobga olgan holda qovushqoq-elastik sterjen uchun avtotebranish jarayoni masalasini ko'rib chiqamiz [5]

$$\sigma = m_1(1 - R^*)\varepsilon, \quad \varepsilon = u_x, \; u = -zw_x \qquad (1)$$

yoki

$$\sigma = -m_1(1 - R^*)zw_x \qquad (2)$$

bu yerda m_1 – elastiklik doimiysi.

Aerodinamik chiziqlilikning ta'sirini hisobga olgan holda, qo'yilayotgan aerodinamik yuk quyidagi shaklni oladi [1]:

$$q = \frac{\chi p_\infty}{c_\infty}\left[V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\right]$$

bu yerda $q = p - p_\infty$, $k = \dfrac{\chi p_\infty}{c_\infty}$

$$q = k\left[V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\right] \qquad (3)$$

Qovushqoq-elastik yupqa devorli konstruksiyalarda avtotebranish jarayonlarining modelini quramiz.

Bunda egilish momenti uchun yassi kesimlar gipotezasini qabul qilib, quyidagi formuladan foydalanamiz [2]:

$$M_x = \int_{-h/2}^{h/2} b(x)\sigma_x z dz \qquad (4)$$

(2) ni (4) ga qo‘yib, quyidagiga ega bo‘lamiz:

$$M_x = -b(x)\left(1-R^*\right)\int_{-h/2}^{h/2} m_1 z w_{xx} z dz = -b(x)\left(1-R^*\right)m_1 w_{xx}\int_{-h\backslash 2}^{h/2} z^2 dz =$$

$$= -b(x)\left(1-R^*\right)m_1 w_{xx}\frac{z^3}{3}\Bigg|_{-h\backslash 2}^{h\backslash 2} = -b(x)\left(1-R^*\right)m_1 w_{xx}\frac{h^3}{12} =$$

$$= -\left(1-R^*\right)m_1\frac{b(x)h^3(x)}{12}w_{xx}$$

$$M_x = -E(1-R^*)J_2 w_{xx} \qquad (5)$$

$b(x)$ – sterjen eni va $h(x)$ – balandligi

$$J_2 = \frac{b(x)h^3(x)}{12}.$$

(5) ni muvozanat tenglamasiga [2] qo‘yib, o‘lchamsiz koordinatalarga o‘tgan holda quyidagiga ega bo‘lamiz:

$$-(1-R^*)\frac{\partial}{\partial x^2}\left[\frac{m_1 J_2^0 h_0}{a^2}g(x)w_{xx}\right] = m_0 F(x)\frac{h_0}{t_1^2}w_{tt} + kV\frac{h_0}{a}w_x + kz\frac{h_0}{t_1}w_t$$

$$-m_1 J_2^0\frac{h_0}{a^4}\left(1-R^*\right)\frac{\partial}{\partial x^2}\left[g(x)w_{xx}\right] = m_0 F(x)\frac{h_0}{t_1^2}w_{tt} + kV\frac{h_0}{a}w_x + kz\frac{h_0}{t_1}w_t$$

$$\left(1-R^*\right)\frac{\partial}{\partial x^2}\left[g(x)w_{xx}\right] + F(x)w_{tt} + Pw_x + \gamma w_t = 0 \qquad (6)$$

bu yerda

$$w = h_0\overline{w},\ x = a\bar{x},\ t = t_1\bar{t},\ m(x) = m_0\overline{F(x)},\ h(x) = h_0\overline{h(x)},\ b(x) = b_0\overline{b(x)},$$

$$J_2 = J_2^0 g(x),\quad g(x) = b(x)h^3(x),\quad J_2^0 = \frac{b_0 h_0^3}{12},$$

$$P = \frac{kVa^3}{m_1 J_2^{(0)}},\quad t_1 = \sqrt{\left(m_0 a^4\right)/\left(m_1 J_2^{(0)}\right)},\quad \gamma = \frac{kza^4}{m_1 J_2^{(0)} t_1},\ F(x) = b(x)h(x)$$

$b(x)=c-a_1x;\ h(x)=1-a_2x;\ c=5$

h_0 – sterjen uchlaridagi balandlik kattaligi, b_0 – sterjen uchlaridagi kenglik kattaligi, m_0 – sterjenning birlik o‘zgaruvchan qimsiga mos keladigan massa qiymati.

Hususiy hosilali chiziqli IDTlar (6), chegaraviy [4] va boshlang‘ich shartlar bilan birgalikda chiziqli qovushqoq-elastik sterjenlar uchun avtotebranuvchan jarayon masalasining matematik modelini ifodalaydi. Tebranishlarning ortib boryotgan amplitudasiga olib keladigan P_{kr} kritik tezlikni topish talab qilinadi.

Taqribiy yechimni Bubnova-Galerkin metodi bilan topamiz. (6) IDT yechimini quyidagi ko‘rinishda olamiz

$$w = \sum_{k=1}^{N} u_k(t)\varphi_k(x) \qquad (7)$$

bu yerda $\varphi_k(x)$ – berilgan chegaraviy shartlarni qanoatlantiruvchi bazis funksiyalar, $u_k(t)$ – aniqlanishi lozim bo‘lgan va vaqtga bog‘liq noma’lum funksiyalar.

$u_k(t)$ noma’lum funksiyalarni topish uchun (7) ni (6) ga qo‘yiladi

$$(1-R^*)\sum_{k=1}^{N}u_k(t)\left[g(x)\varphi_k''(x)\right]'' + F(x)\sum_{k=1}^{N}\ddot{u}_k(t)\varphi_k(x) + P\sum_{k=1}^{N}u_k(t)\varphi_k'(x) + \gamma\sum_{k=1}^{N}\dot{u}_k(t)\varphi_k(x) = 0$$

$\varphi_i(x)$ ga ko‘paytirilib va x bo‘yicha integrallab, quyidagiga ega bo‘linadi:

$$(1-R^*)\sum_{k=1}^{N}u_k(t)\int_0^1\left[g(x)\varphi_k''(x)\right]''\varphi_i(x)dx + \sum_{k=1}^{N}\ddot{u}_k(t)\int_0^1 F(x)\varphi_k(x)\varphi_i(x)dx +$$

$$+ P\sum_{k=1}^{N}u_k(t)\int_0^1\varphi_k'(x)\varphi_i(x)dx + \gamma\sum_{k=1}^{N}\dot{u}_k(t)\int_0^1\varphi_k(x)\varphi_i(x)dx = 0$$

Integrallar uchun belgilashlar kiritib, quyidagi oddiy IDTlar chiziqli sistemasiga kelamiz

$$\sum_{k=1}^{N}\left[a_{ki}\ddot{u}_k(t) + \gamma b_{ki}\dot{u}_k(t) + \omega_{ki}(1-R^*)u_k(t) + Pd_{ki}u_k(t)\right] = 0,\ i=\overline{1,N} \qquad (8)$$

bu yerda

$$a_{ki} = \int_0^1 F(x)\varphi_k(x)\varphi_i(x)dx, \qquad b_{ki} = \int_0^1\varphi_k(x)\varphi_i(x)dx,$$

$$\omega_{ki} = \int_0^1\left[d(x)\varphi_k''(x)\right]''\varphi_i(x)dx, \qquad d_{ki} = \int_0^1\varphi_k'(x)\varphi_i(x)dx,$$

Konstruksiyaning fizik-mexanik parametrlarining keng diapazonda o‘zgarishini hisobga olgan holda $R(t)=A\cdot e^{-\beta t}t^{\alpha-1}$, $A>0$, $\beta>0$, $0<\alpha<1$ Rjanisына–Koltunov yadrosida (8) chiziqli sistemani integrallash analitik almashtirishlarga asoslangan holda [3] sonli usul yordamida bajarildi.

Xulosa. Fizik chiziqli masalalarni tahlil qilish shuni ko‘rsatadiki, kritik tezlik qiymati konstruksiyaning elastik va qovushqoq-elastik holatlarga to‘la bog‘liq.

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