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MATRITSALAR ALGEBRALARIDA LOKAL DIFFERENSIALLASHLAR

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Anotatsiya. So‘nggi yillarda klassik tizimlarning assotsiativ bo‘lmagan analoglari matematika va fizikaning ko‘plab sohalarida qo‘llanilib kelmoqda. Li va Leibniz algebralari kabi ba’zi noassotsiativ algebralar uchun lokal operatorlar, xususan lokal va 2-lokal differensiallashlar tushunchalari ham keng o‘rganila boshladi. Ushbu tushunchalarga aloqador asosiy muammolar har bir lokal (yoki 2-lokal) avtomorfizm va differensiallash oddiy ma’noda avtomorfizm va differensiallash bo‘ladigan shartlarni topish, shuningdek, avtomorfizm bo‘lmagan (mos ravishda differensiallash) lokal va 2-lokal avtomorfizmga ega algebralar sinflarini aniqlashdan iboratdir. Ushbu maqolada matritsalar algebralarida lokal differensiallashlar o‘rganilgan.

Kalit so‘zlar: *Li algebra, differentsiallash, lokal differentsiallash*

Abstract: In recent years, non-associative analogues of classical systems have found applications in many areas of mathematics and physics. For some non-associative algebras, such as Lie and Leibniz algebras, the concepts of local operators, in particular local and 2-local differentiations, have also begun to be widely studied. The main problems associated with these concepts are to find conditions under which every local (or 2-local) automorphism and derivation are automorphisms and derivations in the usual sense, and to determine classes of algebras that have local and 2-local automorphisms that are not automorphisms (derivations, respectively). This article examines local derivations in matrix algebras.

Key words: *Lie algebra, derivation, local derivation*.

Аннотация. В последние годы неассоциативные аналоги классических систем применяются во многих областях математики и физики. Для некоторых неассоциативных алгебр, таких как алгебры Ли и Лейбница, стали широко изучаться понятия локальных операторов, в частности локальных и 2-локальных дифференцирований. Основные проблемы, связанные с этими понятиями, заключаются в нахождении условий, при которых каждый локальный (или 2-локальный) автоморфизм и дифференцирование является автоморфизмом и дифференцированием в простом смысле, а также в определении классов алгебр, не являющихся автоморфизмами (соответственно дифференцированием), с локальным и 2-локальным автоморфизмами. В данной статье изучаются локальные дифференцирования в матричных алгебрах**.**

Ключевые слова: алгебра Ли, дифференцирование, локальное дифференцирование

Kirish. A ba’zi bir algebra (associativ bo‘lmagan). $D:\mathrm{A}\to\mathrm{A}$ chiziqli akslantirish barcha $x,y\in\mathrm{A}$ elementlar uchun $D(xy)=D(x)y+xD(y)$ tengligini qanoatlantirsa, unda bu akslantirish differentsiyallash deyiladi. A chiziqli akslantirish Δ lokal differentsiyallash deyiladi, agar har bir $x\in\mathrm{A}$ uchun D_x differentsiyallash bor bo‘lib bunda A (x elementiga bog‘liq) differentsiyallash ko‘rinishi quyidagicha $\Delta(x)=D_x(x)$. Lokal avtomorfizmning ta’rifi ham xuddi shunday beriladi [1].

Adabiyotlar tahlili. Yuqoridagi tushunchalar R.V. Kadison ishlarida keltirilgan va o‘rganilgan[1]. Shu bilan birgalikda D.R. Larson va A.R. Sourour ishlarida ham keltirilgan [2]. Yuqorida ta’kidlab o‘tilgn ishlar avtomorfizmlar, C^*-algebralar va operator algebralarning differentsiyallashlariga yaqin bo‘lgan akslantirishlarning tavsifiga bag’ishlangan bir qator ishlarga turtki bo‘ldi. D.R. Larson va A.R. Sourourlar $\mathrm{A} = B(X)$ tenglikni isbotlashdi. Sh.A.Ayupov va K.K.Kudaybergenovlar ba’zi chekli o‘lchamli sodda Li va Leibniz algebralarining lokal avtomorfizmlarini o‘rganganlar[5].

Tadqiqot metodologiyasi.

Agar A haqiqiy sonlar maydonida aniqlangan barcha ko‘phadlar algebrasi bo‘lsin. Bu algebrada oddiy hosila amaliy differensiallash operatori bo‘ladi..

$$D(x) = x', \quad A \tag{1}$$

Bu operatorning differensiallovchi operator ekanligi hosilaning xossasidan kelib chiqadi.

1) $D(x+y) = (x+y)' = x' + y' = D(x) + D(y),$

ya’ni

$$D(x+y) = D(x) + D(y)$$

2) $D(\lambda x) = (\lambda x)' = \lambda x' = \lambda D(x)$

ya’ni

$$D(\lambda x) = \lambda D(x)$$

3) $D(xy) = (xy)' = x'y + y'x = D(x)y + xD(y),$

na’ni

$$D(xy) = D(x)y + xD(y).$$

Agar A bazi bir algebra, $P(A)$ bu algebraning idempotentler to‘plami va $D: A \to A$ differentsiyallash bo‘lsin. U holda,quydagi xossalar o‘rinli:

a) agar $e \in P(A)$ bo‘lsa, unda $eD(e)e = 0;$

b) agar $e \in P(A)$ va $eD(e) = D(e)e$ bo‘lsa, unda $D(e) = 0;$

Isboti. a) Mayli $e \in P(A)$ bo‘lsin. Unda $e = e^2$ ekanligidan

$$D(e) = D(e^2) = D(e)e + eD(e),$$

ya’ni

$$D(e) = D(e)e + eD(e).$$

Bu tenglikning o‘ng tamonini e ga ko‘paytirsak

$$D(e)e = D(e)e^2 + eD(e)e,$$

ya’ni

$$D(e)e = D(e)e + eD(e)e$$

Bundan

$$eD(e)e = 0.$$

b) Mayli $e \in P(A)$ va $eD(e) = D(e)e$ bo‘lsin.

Unda,

$$D(e) = D(e)e + eD(e) = D(e)e + D(e)e =$$
$$= 2D(e)e = 2\left[D(e)e\right]e = 2\left[eD(e)\right]e = 2eD(e)e = 0,$$

ya'ni $D(e)=0$.

Mayli $D: A \to A$ differentsiyallash, $x \in A,\ n \in N$. Unda

$$D(x^n)=\sum_{k=1}^{n} C_n^k x^k\, D(x)\, x^{n-k-1}$$

tengligi o'rinli.

$n=2$ da

$$D(x^2)=\sum_{k=1}^{2} C_2^k x^k\, D(x)\, x^{1-k} = xD(x)+D(x)x.$$

Bu tenglik differentsiyallash ayniyatidan ko'ra o'rinli.

$n=m$ da o'rinli bo'lsin, ya'ni

$$D(x^m)=\sum_{k=1}^{m} x^k\, D(x)\, x^{m-k-1}$$

$n=m+1$ da isbotlaymiz.

$$D(x^{m+1})=D(x^m x)=D(x^m)x+x^m D(x)=$$

$$=(\sum_{k=1}^{m} x^k\, D(x)\, x^{m-k-1})x+x^m D(x)=$$

$$=\sum_{k=1}^{m} x^k\, D(x)\, x^{m-k}+x^m D(x)=\sum_{k=1}^{m+1} x^k\, D(x)\, x^{(m+1)-k-1}.$$

Demak

$$D(x^n)=\sum_{k=1}^{n} C_n^k x^k\, D(x)\, x^{n-k-1}.$$

Olingan natijalar va muhokama

Xulosa. Teorema. Meyli R kommutativ algebra bo'lsin. Agar $\Phi: M_n(R) \to M_n(R)$ operatori R-chiziqli bo'lib, barcha $P \in M_n(R)$ idempotentler uchun $\Phi(P)^2=\Phi(P)$ tengligini qanoatlantirsa, unda Φ Jordan gomomorfizmi bo'ladi.

Isboti. E_{ij} orqali $M_n(R)$ algebraning matritsaviy birliklarini belgilaymiz. $M_n(R)$ algebraning har bir $X \in M_n(R)$ elementi

$$X=\sum_{i,j=1}^{n} \lambda_{ij} E_{ij}, \qquad \lambda_{ij} \in R, \quad i,j=\overline{1,n}$$

ko'rinishda bo'lganlikdan

$$\Phi(E_{ij}E_{kl}+E_{kl}E_{ij})=\Phi(E_{ij})\Phi(Ee_{kl})+\Phi(E_{kl})\Phi(E_{ij}) \qquad (2)$$

tengligini ko'rsatish yetarli.

1-hol. $i=j,\ k=l$. Unda

$$\Phi(E_{ii}E_{kk}+E_{kk}E_{ii})=\Phi(E_{ii})\Phi(Ee_{kk})+\Phi(E_{kk})\Phi(E_{ii})$$

Agar $i=k$ bo'lsa, unda

$$\Phi(E_{ii})=\Phi(E_{ii})^2$$

E_{ii} idempotent bo'lganligi uchun bu tenglik teorema shartidan kelib chiqadi.

Endi $i \neq k$ bo‘lsin. Unda $E_{ii}\,E_{kk} = 0$

Demak

$$\Phi(E_{ii})\Phi(E_{kk}) + \Phi(E_{kk})\Phi(E_{ii}) = 0 \qquad (3)$$

tengligining o‘rinli ekenligini ko‘rsatish kerak. $P_1 = E_{ii} + E_{kk}$ bo‘lsin. Unda P_1-proektor bo‘ladi. Shu jumladan

$$\Phi(E_{ii}) + \Phi(E_{kk}) = \Phi(E_{ii} + E_{kk}) = \Phi(P_1) = \Phi(P_1)^2 =$$

$$= \left(\Phi(E_{ii}) + \Phi(E_{kk})\right)^2 = \Phi(E_{ii})^2 + \Phi(E_{kk})^2 + \Phi(E_{ii})\Phi(E_{kk}) +$$

$$+\Phi(E_{kk})\Phi(E_{ii}) = \Phi(E_{ii}) + \Phi(E_{kk}) + \Phi(E_{ii})\Phi(E_{kk}) + \Phi(E_{kk})\Phi(E_{ii}),$$

ya’ni

$$\Phi(E_{ii}) + \Phi(E_{kk}) + \Phi(E_{kk})\Phi(E_{kk}) = 0$$

Endi $i \neq j$ bo‘lganda

$$\Phi(E_{ij})^2 = 0$$

ekanligini isbotlaymiz.

$$P_2 = E_{ii} + E_{ij}, \quad P_3 = E_{ii} - E_{ij}$$

$$P_2^2 = (E_{ii} + E_{ij})^2 = E_{ii}^{\,2} + E_{ii}E_{ij} + E_{ij}E_{ii} + E_{ij}E_{ij} =$$

$$= E_{ii} + E_{ii} = P_1, \quad P_2^2 = P_2$$

shunday qilib, $P_3^2 = P_2$. Demak, P_2, P_3 – proektorlar.

Bundan

$$2\Phi(E_{ii}) = \Phi(E_{ii} + E_{ij}) + \Phi(E_{ii} - E_{ij}) = \Phi(E_{ii} + E_{ij})^2 + \Phi(E_{ii} - E_{ij})^2 =$$

$$= \Phi(E_{ii}) + \Phi(E_{ii})\Phi(E_{ij}) + \Phi(E_{ij})\Phi(E_{ii}) + \Phi(E_{ij})^2 +$$

$$+\Phi(E_{ii}) - \Phi(E_{ii})\Phi(E_{ij}) - \Phi(E_{ij})\Phi(E_{ii}) + \Phi(E_{ij})^2 = 2\Phi(E_{ii}) + 2\Phi(E_{ij})^2$$

Bundan $\Phi(E_{ij})^2 = 0$.

2-hol. $i = j$, $k \neq l$ bo‘lsin.

Avval $i = k$ holini ko‘ramiz. Unda

$$2\Phi(e_{il}) = \Phi(e_{ii})\Phi(e_{il}) + \Phi(e_{il})\Phi(e_{ii}) \qquad (4)$$

ekanligini ko‘rsatish kerak. $P_4 = e_{ii} + e_{il}$ proektor bo‘lganligidan,

$$\Phi(e_{ii}) + \Phi(e_{il}) = \Phi(P_4) = \left(\Phi(e_{ii}) + \Phi(e_{il})\right)^2 =$$

$$= \Phi(e_{ii}) + \Phi(e_{il})^2 + \left(\Phi(e_{ii})\Phi(e_{il}) + \Phi(e_{il})\Phi(e_{ii})\right).$$

Endi (4) tenglikni qo‘llasak, (5) kelib chiqadi. Shunday qilib

$$\Phi(e_{ki}) = \Phi(e_{ii})\Phi(e_{ki}) + \Phi(e_{ki})\Phi(e_{ii}), \qquad (5)$$

3-hol. $k \neq i$, $l \neq i$. Unda $e_{ii}\,e_{kl} + e_{kl}\,e_{ii} = 0$ bo‘lganligidan,

$$\Phi(e_{ii})\Phi(e_{kl}) + \Phi(e_{kl})\Phi(e_{ii}) = 0$$

ekanligini ko‘rsatamiz. $P_5 = e_{kk} + e_{ii} + e_{kl}$ bo‘lsin. Unda

$$P_5^2=\left(e_{kk}+e_{ii}+e_{kl}\right)^2=\left(e_{kk}+e_{ii}+e_{kl}\right)\left(e_{kk}+e_{ii}+e_{kl}\right)=$$
$$=e_{kk}+e_{kk}e_{kl}+e_{ii}=e_{kk}+e_{ii}+e_{kl}=P_5,$$

ya'ni P_5-idempotent.

Bundan

$$\Phi\left(e_{kk}\right)+\Phi\left(e_{ii}\right)+\Phi\left(e_{kl}\right)=\Phi\left(P_5\right)=\left(\Phi\left(e_{kk}\right)+\Phi\left(e_{ii}\right)+\Phi\left(e_{kl}\right)\right)^2=$$
$$=\Phi\left(e_{kk}\right)+\Phi\left(e_{ii}\right)+\Phi\left(e_{kl}\right)^2+\Phi\left(e_{kk}\right)\Phi\left(e_{kl}\right)+\Phi\left(e_{kl}\right)\Phi\left(e_{kk}\right)+$$
$$+\Phi\left(e_{ii}\right)\Phi\left(e_{kl}\right)+\Phi\left(e_{kl}\right)\Phi\left(e_{ii}\right).$$

Bu tenglikga (3), (4) va (5) larni qo'llasak, (6) tenglik kelib chiqadi.

4-hol. $i\neq j,\ k\neq l$. Agar $i=k,\ j=l$ bo'lsa, unda (4) formuladan (2) kelib chiqadi.

Agar $j\neq l$ bo'lsa,

$$\Phi\left(e_{ij}\right)\Phi\left(e_{il}\right)+\Phi\left(e_{il}\right)\Phi\left(e_{ij}\right)=0 \qquad (7)$$

ekenligini ko'rsatish kerak.

$P_6=e_{ii}+e_{ij}+e_{il}$ proektor bo'lganligidan,

(7) tenglik $\Phi\left(P_6\right)=\Phi\left(P_6\right)^2$, (4) va (5) tengliklardan kelib chqadi.

5-hol. $i\neq j,\ k\neq l,\ i\neq k$. Bunda quydagi 4 holatni ko'ramiz:

a) $i=l,\quad j=k;$

b) $i=l,\quad j\neq k;$

s) $i\neq l;\quad j=k;$

d) $i\neq l;\quad j\neq k.$

a) bo'lganda,

$$\Phi\left(e_{ij}\right)\Phi\left(e_{ji}\right)+\Phi\left(e_{ji}\right)\Phi\left(e_{ij}\right)=\Phi\left(e_{ii}\right)+\Phi\left(e_{jj}\right)$$

ekanligini ko'rsatimiz. $P_7=\frac{1}{2}\left(e_{ii}+e_{jj}-e_{ij}-e_{ji}\right)$ proektor bo'lganligidan,

$$2\left(\Phi\left(e_{ii}\right)+\Phi\left(e_{jj}\right)-\Phi\left(e_{ij}\right)-\Phi\left(e_{ji}\right)\right)=4\Phi\left(P_7\right)=4\Phi\left(P_7\right)^2=$$
$$=\Phi\left(e_{ii}\right)+\Phi\left(e_{jj}\right)+\Phi\left(e_{ij}\right)^2+\Phi\left(e_{ji}\right)^2+$$
$$+\Phi\left(e_{ii}\right)\Phi\left(e_{jj}\right)+\Phi\left(e_{jj}\right)\Phi\left(e_{ii}\right)-\left(\Phi\left(e_{ii}\right)\Phi\left(e_{ij}\right)+\Phi\left(e_{ij}\right)\Phi\left(e_{ii}\right)\right)-$$
$$-\left(\Phi\left(e_{ii}\right)\Phi\left(e_{ji}\right)+\Phi\left(e_{ji}\right)\Phi\left(e_{ii}\right)\right)-\left(\Phi\left(e_{jj}\right)\Phi\left(e_{ij}\right)+\Phi\left(e_{ij}\right)\Phi\left(e_{jj}\right)\right)-$$
$$-\left(\Phi\left(e_{jj}\right)\Phi\left(e_{ji}\right)+\Phi\left(e_{ji}\right)\Phi\left(e_{jj}\right)\right)+\left(\Phi\left(e_{ij}\right)\Phi\left(e_{ji}\right)+\Phi\left(e_{ji}\right)\Phi\left(e_{ij}\right)\right).$$

endi (2) tenglik (3), (4), (5) va (6) lardan kelib chiqadi.

b) holda $P_8=e_{ii}+e_{ij}+e_{ki}+e_{kj}$ proektorini olamiz. $\Phi\left(P_8\right)=\Phi\left(P_8\right)^2$ tengligidan (2) kelib chiqadi.

s) holati b-bilan bir xil. d) holda $P_9=e_{ii}+e_{kk}+e_{ij}+e_{kl}$ ga proektoridan foydalansak kelib chiqadi.

Teorema isbotlandi.

Foydalanilgan adabiyotlar ro‘yxati.

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