UNIVERSAL JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES
ISSN 2992 – 8788
https://sr-journals.org/index.php/UJMTCS

Aralash uchburchakli sohada parabolik-giperbolik tipdagi uchinchi tartibli tenglama uchun bitta chegaraviy masalani tadqiq etish

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Annotatsiya. Ushbu maqolada koordinatalar tekisligining aralash uchburchakli sohasida qaraluvchi uchinchi tartibli parabolik-giperbolik tipdagi differensial tenglama uchun chegaraviy masala tadqiq etilgan. Tadqiqot davomida masalaning matematik qo‘yilishi aniq shakllantirilib, tenglama uchun mos chegaraviy hamda ulash shartlari berilgan. Aralash tipdagi tenglamalarning o‘ziga xos xususiyatlari hisobga olinib, masalani yechishning analitik usullari ishlab chiqilgan. Xususan, yechimni qurish usuli yordamida masala integral tenglamaga keltirilgan va Volterra tipidagi integral tenglama hosil qilingan. Shuningdek, tur o‘zgarish chizig‘idagi ulash shartlari yordamida yechimning uzluksizligi va mosligi ta’minlangan. Maqolada qo‘yilgan chegaraviy masalaning mavjudlik va yagonalik shartlari teorema ko‘rinishida isbotlangan. Olingan natijalar aralash tipdagi yuqori tartibli differensial tenglamalar nazariyasini yanada rivojlantirishda muhim ahamiyat kasb etadi. Mazkur ish kelgusida murakkab chegaraviy masalalarni tadqiq etishda nazariy asos sifatida xizmat qiladi.

Kalit so‘zlar: Differensial va integral tenglamalar, yechimni qurish usuli, davom etish usuli, chegaraviy masala, parabolik-giperbolik tip, yechimning yagonaligi.

Аннотация. В данной статье исследуется краевая задача для параболическо-гиперболического дифференциального уравнения третьего порядка, рассматриваемого в смешанной треугольной области координатной плоскости. В ходе исследования была четко сформулирована математическая постановка задачи и даны соответствующие граничные и связные условия для уравнения. Разработаны аналитические методы решения задачи с учетом специфики уравнений смешанного типа. В частности, задача была сведена к интегральному уравнению с использованием метода построения решения, и получено интегральное уравнение типа Вольтерры. Также непрерывность и согласованность решения были обеспечены с помощью условий связности на линии смены типа. Условия существования и единственности краевой задачи, поставленной в статье, доказаны в виде теоремы. Полученные результаты имеют большое значение для дальнейшего развития теории дифференциальных уравнений высшего порядка смешанного типа. Данная работа послужит теоретической основой для изучения комплексных краевых задач в будущем.

Ключевые слова: дифференциальные и интегральные уравнения, метод построения решения, метод продолжения, краевая задача, параболическо-гиперболический тип, единственность решения.

Abstract. This article studies the boundary value problem for a third-order parabolic-hyperbolic differential equation considered in a mixed triangular region of the coordinate plane. During the research, the mathematical formulation of the problem was clearly formulated, and appropriate boundary and connection conditions for the equation were given. Analytical methods for solving the problem were developed taking into account the specific features of mixed-type equations. In particular, the problem was reduced to an integral equation using the solution construction method and an integral equation of the Volterra type was obtained. Also, the continuity and consistency of the solution were ensured using the connection conditions on the type change line. The existence and uniqueness conditions of the boundary value problem posed in the article were proved in the form of a theorem. The results obtained are of great importance in the further development of the theory of

mixed-type higher-order differential equations. This work will serve as a theoretical basis for the study of complex boundary value problems in the future.

Keywords: Differential and integral equations, solution construction method, continuation method, boundary value problem, parabolic-hyperbolic type, uniqueness of solution.

Ushbu maqolada xOy tekislikning aralash uchburchakli G sohasida uchinchi tartibli parabolik-giperbolik tipdagi

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+c\right)(Lu)=0 \tag{1}$$

tenglama uchun bitta chegaraviy masala qo‘yiladi, bunda

$$Lu=\begin{cases}u_{1xx}-u_{1y}, & (x,y)\in G_1,\\ u_{ixx}-u_{iyy}, & (x,y)\in G_i \quad (i=2,3),\end{cases} \quad c\in R,\ u(x,y)=u_i(x,y),\ (x,y)\in G_i \quad (i=1,2,3),$$

$$G=G_1\cup G_2\cup G_3\cup J_1\cup J_2,\ G_1=\{(x,y)\in R^2: 0<x<1,\ 0<y<1\},$$

$$G_2=\{(x,y)\in R^2: -1<y<0,\ -y<x<1\},\ G_3=\{(x,y)\in R^2: -1<x<0,\ -x<y<1\},$$

$$J_1=\{(x,y)\in R^2: y=0,\ 0<x<1\},\ J_2=\{(x,y)\in R^2: x=0,\ 0<y<1\},$$

ya’ni G_1 uchlari $A(0;0)$, $B(1;0)$, $B_0(1,1)$, $A_0(0,1)$ nuqtalarda bo‘lgan to‘g’ri to‘rtburchak; G_2 uchlari A, B, $C(1,-1)$ nuqtalarda bo‘lgan uchburchak; G_3 uchlari A, $D(-1,1)$, A_0 nuqtalarda bo‘lgan uchburchak; J_1 uchlari A, B nuqtalarda bo‘lgan ochiq kesma; J_2 uchlari A, A_0 nuqtalarda bo‘lgan ochiq kesma.

Bizga G_i $(i=2,3)$ sohalarni quyidagi ko‘rinishda yozib olish kerak bo‘ladi (ulardan biz kelgusida foydalanamiz): $G_2=G_{21}\cup G_{22}\cup BE_1$, $G_3=G_{31}\cup G_{32}\cup A_0E_2$, bunda G_{21} – uchlar A, B, $E_1\left(\frac{1}{2},-\frac{1}{2}\right)$ nuqtalarda bo‘lgan uchburchak; G_{22} – uchlari B, C, E_1 nuqtalarda bo‘lgan uchburchak; G_{31} – uchlari A, A_0, $E_2\left(-\frac{1}{2},-\frac{1}{2}\right)$ nuqtalarda bo‘lgan uchburchak; G_{32} – uchlari A_0, D, E_2 nuqtalarda bo‘lgan uchburchak; BE_1 – uchlari B, E_1 nuqtalarda bo‘lgan ochiq kesma; A_0E_2 – uchlari A_0, E_2 nuqtalarda bo‘lgan ochiq kesma, ya’ni

$$G_{21}=\left\{(x,y)\in R^2: -\frac{1}{2}<y<0,\ -y<x<y+1\right\},\ G_{22}=\left\{(x,y)\in R^2: \frac{1}{2}<x<1,\ -x<y<x-1\right\},$$

$$G_{31}=\left\{(x,y)\in R^2: -\frac{1}{2}<x<0,\ -x<y<x+1\right\},\ G_{32}=\left\{(x,y)\in R^2: \frac{1}{2}<y<1,\ -y<x<y-1\right\},$$

$$BE_1=\left\{(x,y)\in R^2: \frac{1}{2}<x<1,\ y=x-1\right\},\ A_0E_2=\left\{(x,y)\in R^2: -\frac{1}{2}<x<0,\ y=x+1\right\}.$$

Masala: Shunday $u(x,y)$ funksiya topilsinki, u 1) $\overline{G}$ yopiq sohada uzluksiz; 2) (1) tenglamani ochiq G sohada $x\neq 0,\ y\neq 0$ bolganda qanoatlantiradi; 3) quyidagi chegaraviy shartlarni qanoatlantiradi:

$$u(1,y)=\varphi_1(y),\quad 0\le y\le 1, \tag{2}$$

$$u(1,y)=\varphi_2(y),\ -1\le y\le 0, \tag{3}$$

$$u|_{AE_1}=\psi_1(x),\ 0\le x\le\frac{1}{2}, \tag{4}$$

$$u|_{AE_2}=\psi_2(x),\ -\frac{1}{2}\le x\le 0, \tag{5}$$

$$u_3(x,1) = f_1(x),\ -1 \le x \le 0, \tag{6}$$

$$\left.\frac{\partial u_2}{\partial n}\right|_{AC} = \psi_3(x),\ 0 \le x \le 1, \tag{7}$$

$$\left.\frac{\partial u_3}{\partial n}\right|_{AD} = \psi_4(x),\ -1 \le x \le 0; \tag{8}$$

va 4) tur o‘zgarish chizig’ida ushbu ulash shartlarini qanoatlantiradi:

$$u_1(x,0) = u_2(x,0) = \tau_1(x),\ 0 \le x \le 1, \tag{9}$$

$$u_{1y}(x,0) = u_{2y}(x,0) = \nu_1(x),\ 0 \le x \le 1, \tag{10}$$

$$u_{1yy}(x,0) = u_{2yy}(x,0) = \mu_1(x),\ 0 \le x \le 1, \tag{11}$$

$$u_1(0,y) = u_3(0,y) = \tau_2(y),\ 0 \le y \le 1, \tag{12}$$

$$u_{1x}(0,y) = u_{3x}(0,y) = \nu_2(y),\ 0 \le y \le 1, \tag{13}$$

$$u_{1xx}(0,y) = u_{3xx}(0,y) = \mu_2(y),\ 0 \le y \le 1. \tag{14}$$

Bunda $\varphi_i\ (i=1,2), \psi_j\ (j=1,2,3,4), f_1$ – berilgan yetarli silliq funksiyalar, $\tau_i, \nu_i, \mu_i\ (i=1,2)$ lar esa hozircha noma’lum yetarli silliq funksiyalar, shu bilan birga $\tau_1(1) = \varphi_1(0) = \varphi_2(0)$, $\psi_2(-1) = f_1(-1)$, $\varphi_2(-1) = \psi_1(1)$, $\psi_3(0) = \psi_4(0)$ kelishish shartlari bajariladi, $n-$ ushbu $x+y=0$ to‘g’ri chiziqqa o‘tkazilgan ichki normal.

Teorema. Agar $\varphi_1 \in C^3[0,1]$, $\varphi_2 \in C^3[-1,0]$, $\psi_1 \in C^3\left[\frac{1}{2},1\right]$, $\psi_2 \in C^3\left[-1,-\frac{1}{2}\right]$, $\psi_3 \in C^2[0,1]$, $\psi_4 \in C^2[-1,0]$, $f_1 \in C^3[-1,0]$ bo‘lib, shu bilan birga $\tau_1(1) = \varphi_2(0) = \varphi_1(0)$, $f_1(-1) = \psi_2(-1)$, $\tau_1'(1) = \nu_3(0)$, $\varphi_2(-1) = \psi_1(1)$, $\tau_1''(1) = \varphi_2''(0) + \sqrt{2}\psi_3'(1)\exp\left(-\frac{c}{b}\right)$, $\psi_3(0) = \psi_4(0)$ kelishish shartlari bajarilsa, u holda M_{11c}^3 masala yagona yechimga ega bo‘ladi.

Isbot. Teoremani yechimni qurish usuli yordamida isbot qilamiz. Buning uchun (1) tenglamani

$$u_{1xx} - u_{1y} = \omega_1(x-y)e^{-cy},\ (x,y) \in G_1, \tag{15}$$

$$u_{ixx} - u_{iyy} = \omega_i(x-y)e^{-cy},\ (x,y) \in G_i\ \ (i=2,3), \tag{16}$$

ko‘rinishda yozib olamiz, bunda $u(x,y) = u_i(x,y),\ (x,y) \in G_i\ \ (i=\overline{1,3})$ belgilash kiritildi, shuningdek, $\omega_i(x-y)\ (i=\overline{1,3})$ – hozircha noma’lum yetarlicha silliq funksiyalar.

$G_i\ \ (i=2,3)$ sohalarning yuqorida yozilgan ko‘rinishlarini e’tiborga olib, (16) $(i=2,3)$ tenglamani

$$u_{ikxx} - u_{ikyy} = \omega_{ik}(x-y)e^{-cy},\ (x,y) \in G_{ik}\ \ (i=2,3;\ k=1,2) \tag{17}$$

ko‘rinishda yozib olamiz, bunda $u_i(x,y) = u_{ik}(x,y)$, $\omega_i(x-y) = \omega_{ik}(x-y)$, $(x,y) \in G_{ik}$ $(i=2,3;\ k=1,2)$ belgilashlar kiritildi.

Ushbu belgilashni kiritamiz:

$$\omega_1(x-y) = \begin{cases} \omega_{11}(x-y), & -1 \le x-y \le 0, \\ \omega_{12}(x-y), & 0 \le x-y \le 1. \end{cases}$$

(15) tenglamada $y \to 0$ deb limitga o‘tib, ba’zi almashtirishlardan so‘ng

$$\omega_{12}(x-y) = \tau_1''(x-y) - \nu_1(x-y),\ 0 \le x-y \le 1 \tag{18}$$

ni topamiz.

(15) tenglamada $x \to 0$ deb limitga o‘tib esa ushbu munosabatni olamiz:

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$$\mu_2(y)-\tau_2'(y)=\omega_{11}(-y)e^{-cy},\ 0\le y\le 1. \tag{19}$$

va yana (17) $(i=3;\ k=1)$ tenglamada $x\to 0$ deb limitga o‘tsak,

$$\mu_2(y)-\tau_2''(y)=\omega_{31}(-y)e^{-cy},\ 0\le y\le 1 \tag{20}$$

ni olamiz. (19) va (20) munosabatlardan $\mu_2(y)$ funksiyani yo‘qotib,

$$\omega_{11}(-y)=\left[\tau_2''(y)-\tau_2'(y)\right]e^{cy}+\omega_{31}(-y),\ 0\le y\le 1 \tag{21}$$

ga ega bo‘lamiz.

Endi (15) tenglamaning (2), (9), (12) shartlarni qanoatlantiruvchi yechimini yozib olamiz:

$$u_1(x,y)=\int_0^y\tau_2(\eta)G_\xi(x,y;0,\eta)d\eta-\int_0^y\varphi_1(\eta)G_\xi(x,y;1,\eta)d\eta+$$

$$+\int_0^1\tau_1(\xi)G(x,y;\xi,0)d\xi-\int_0^y e^{-c\eta}d\eta\int_0^\eta\omega_{11}(\xi-\eta)G(x,y;\xi,\eta)d\xi-$$

$$-\int_0^y e^{-c\eta}d\eta\int_\eta^1\omega_{12}(\xi-\eta)G(x,y;\xi,\eta)d\xi,$$

bunda $\left.\begin{matrix}G(x,y;\xi,\eta)\\N(x,y;\xi,\eta)\end{matrix}\right\}=\dfrac{1}{2\sqrt{\pi(y-\eta)}}\sum\limits_{n=-\infty}^{+\infty}\left\{\exp\left[-\dfrac{(x-\xi-2n)^2}{4(y-\eta)}\right]\mp\exp\left[-\dfrac{(x+\xi-2n)^2}{4(y-\eta)}\right]\right\}$ lar (15) tenglama uchun mos ravishda birinchi va ikkinchi chegaraviy masalalarning Grin funksiyalari.

Bu yechimni x bo‘yicha differensiallab va x ni nolga intiltirib, (18), (19) va (21) larga ko‘ra uzoq hisoblashlardan so‘ng, noma’lum $\tau_2'(y)$ funksiyaga nisbatan ushbu ikkinchi jins Volterr tipidagi integral tenglamaga kelamiz:

$$\tau_2'(y)+\int_0^y K(y,\eta)\tau_2'(\eta)d\eta=g(y), \tag{22}$$

bunda $K(y,\eta)$, $g(y)$ lar ma’lum funksiyalar, shuningdek, $K(y,\eta)$ yadro kuchsiz $\left(\dfrac{1}{2}\right)$ maxsuslikka ega, o‘ng tomondagi $g(y)$ funksiya esa uzluksiz. U holda (22) tenglama uzluksiz funkjsiyalar sinfida yagona yechimga ega bo‘ladi. (22) tenglamani yechib, $\tau_2'(y)$ funksiyani, va demak, $\nu_2(y)$, $\omega_{11}(bx-ay)$, $u_1(x,y)$, $u_{31}(x,y)$ funksiyalarni ham topiladi.

Agar $u_{31}(x,x+1)=h_2(x)$ deb belgilash kiritsak, u holda $u_{32}(x,y)$ funksiyani aniqlash uchun

$$u_{32}(x,x+1)=h_2(x) \tag{23}$$

shartni olamiz.

keyin esa $\nu_4(x)$ funksiyani topamiz:

$$\nu_4(x)=h_2'\left(\frac{x}{2}\right)-f_1'(x)+\int_0^{\frac{x+2}{2}}e^{-c\eta}\omega_{32}(x+1-2\eta)d\eta,\ -1\le x\le 0.$$

Shunday qilib, biz $u_{32}(x,y)$ funksiyani ham aniqladik va demak, qo‘yilgan masalaning yechimi to‘liq topildi.

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