NILRADIKALI VITT ALGEBRASI BO‘LGAN YECHILUVCHAN LI ALGEBRASINING LOKAL DIFFERENSIALLANISHI

Yuldashev Inomjon G‘ulomboy o‘g‘li
Nukus davlat pedagogika instituti Matematika o‘qitish metodikasi kafedrasi assistent o’qituvchisi i.yuldashev1990@mail.ru
ORCID: 0009-0004-4268-5420

UDK 517.98

ANOTATSIYA. *Xarakteristikasi nolga teng algebraik yopiq maydon ustida chekli o‘lchamli Li algebralarida lokal va 2-lokal differensiallashlar va avtomorfizmga oid yurtimizda bir qator ilmiy izlanishlar olib borilib, dastlabki natijalar Sh.A.Ayupov, K.K.Kudaybergenov, I.S.Raximovlarga tegishlidir[3]. Ular tomonidan yarim sodda Li algebrasidagi har bir 2-lokal differensiallash oddiy ma’noda differensiallash ekanligi va o‘lchami ikkidan katta ixtiyoriy nilpotent Li algebrasida differensiallash bo‘lmagan 2-lokal differensiallash mavjudligini isbotlangan. Keyinchalik Sh.A.Ayupov va K.K.Kudaybergenovlar yarim sodda Li algebralaridagi har bir lokal differensiallash ham oddiy ma’noda differensiallash ekanligini isbotladilar va differensiallash bo‘lmagan lokal differensiallashi mavjud bo‘lgan chekli o‘lchamli nilpotent Li algebralariga misollar keltirdilar[4].*

Ushbu maqolada Nilradikali Vitt algebrasi bo‘lgan yechiluvchan Li algebrasining lokal differensiallanishi tasnifini keltiramiz.

Kalit so‘zlar: *Li algebra, differentsiallash, lokal differentsiallash, ichki differensiyallash, 2-lokal differensiallashlar, yechiluvchan Li algebrasi, Witt algebrasi.*

Abstract: *In our country, a number of scientific studies have been conducted on local and 2-local differentiations and automorphisms in finite-dimensional Lie algebras over algebraically closed fields of characteristic zero, and the first results belong to Sh. A. Ayupov, K. K. Kudaybergenov and I. S. Rakhimov [3]. They proved that every 2-local differentiation in a semisimple Lie algebra is a differentiation in the usual sense, and that there exists a 2-local differentiation that is not a differentiation in an arbitrary nilpotent Lie algebra of dimension greater than two. Later, Sh. A. Ayupov and K. K. Kudaybergenov proved that every local differentiation in semisimple Lie algebras is also a differentiation in the usual sense, and gave examples of finite-dimensional nilpotent Lie algebras that have a local differentiation that is not a differentiation [4]. In this paper we present a classification of local differentiations of a solvable Lie algebra, which is a Witt algebra with nilradicals.*

Key words: *Lie algebra, differentiation, local differentiation, inner differentiation, 2-local differentiations, solvable Lie algebra, Witt algebra.*

Аннотация: *В нашей стране проведен ряд научных исследований по локальным и 2-локальным дифференциациям и автоморфизмам в конечномерных алгебрах Ли над алгебраически замкнутыми полями нулевой характеристики, и первые результаты принадлежат Ш.А. Аюпову, К.К. Кудайбергенову и И.С.Рахимову [3]. Они доказали, что всякое 2-локальное дифференцирование в полупростой алгебре Ли является дифференцированием в обычном смысле, и что существует 2-локальное дифференцирование, не являющееся дифференцированием в произвольной нильпотентной алгебре Ли размерности больше двух. Позднее Ш.А. Аюпов и К.К. Кудайбергенов доказали, что всякое локальное дифференцирование в полупростых алгебрах Ли также является дифференцированием в обычном смысле, и привели примеры конечномерных нильпотентных алгебр Ли, имеющих локальное дифференцирование, не являющееся дифференцированием[4]. В этой статье мы представляем классификацию локальных дифференциаций разрешимой алгебры Ли, которая является алгеброй Витта с нильрадикалами.*

Ключевые слова: *Алгебра Ли, дифференцирование, локальное дифференцирование, внутреннее дифференцирование, 2-локальные дифференцирования, разрешимая алгебра Ли, алгебра Витта*.

Kirish. A ba'zi bir algebra (associativ bo'lmagan). $D:\mathrm{A}\to\mathrm{A}$ chiziqli akslantirish barcha $x,y\in\mathrm{A}$ elementlar uchun $D(xy)=D(x)y+xD(y)$ tengligini qanoatlantirsa, unda bu akslantirish differentsiyallash deyiladi. A chiziqli akslantirish Δ lokal differentsiyallash deyiladi, agar har bir $x\in\mathrm{A}$ uchun D_x differentsiyallash bor bo'lib bunda A (x elementiga bog'liq) differentsiyallash ko'rinishi quyidagicha $\Delta(x)=D_x(x)$. Lokal avtomorfizmning ta'rifi ham xuddi shunday beriladi [1].

Adabiyotlar tahlili. Yuqoridagi tushunchalar R.V. Kadison ishlarida keltirilgan va o'rganilgan[1]. Shu bilan birgalikda D.R. Larson va A.R. Sourour ishlarida ham keltirilgan [2]. Yuqorida ta'kidlab o'tilgn ishlar avtomorfizmlar, C^*-algebralar va operator algebralarning differentsiyallashlariga yaqin bo'lgan akslantirishlarning tavsifiga bag'ishlangan bir qator ishlarga turtki bo'ldi. D.R. Larson va A.R. Sourourlar $\mathrm{A}=B(X)$ tenglikni isbotlashdi. Sh.A.Ayupov va K.K.Kudaybergenovlar ba'zi chekli o'lchamli sodda Li va Leibniz algebralarining lokal avtomorfizmlarini o'rganganlar[5].

Tadqiqot metodologiyasi. Yechiluvchan Li algebralari

L Li algebrasi bo'lsin. Quyidagi markaziy qatorlarni qaraylik:

$$\mathrm{L}^1=\mathrm{L};\quad \mathrm{L}^{k+1}=[\mathrm{L}^k,\mathrm{L}],\quad k\geq 1,$$

$$\mathrm{L}^{[1]}=\mathrm{L},\quad L^{[s+1]}=[L^{[s]},\mathrm{L}^{[s]}],\quad s\geq 1.$$

L Li algebrasi nilpotent deyiladi (mos ravishda, yechiluvchan), agar shunday $p \in \mathrm{N}$ ko'rinishdagi $\mathrm{L}^p = 0$ (mos ravishda, $\mathrm{L}^{[p]} = 0$). Bunda k eng kichik butun son $\mathrm{L}^k = 0$ tengligi L ning nilindeksi (yoki nilpotentlik kllasi) deyiladi. [6]

W_n^+ ning lokal differensiallari. $n \geq 5$ va W_n^+ $\{e_0, e_1, e_2, ..., e_n\}$ bazisga ega$(n+1)$ –o'lchamli yechiluvchan Li algebrasi bo'lsin.

$$[e_i, e_j] = (j-i)e_{i+j}, \qquad 0 \leq i, j \leq n,\ i+j \leq n.$$

$$W_n = [W_n^+, W_n^+] = span\{e_1, e_2, ..., e_n\}$$

orqali n–o'lchamli filiform Li algebrasini belgilaylik. Bu W_n algebra Vitt algebrasi deyiladi. [7, teorema 4.1] ga ko'ra W_n^+ning barcha differensiali ichki differensial bo'ladi .

$E_{i,j}$, $i, j \in \{0,1,...,n\}$ bazis vektor orqali quyidagicha aniqlangan W_n^+ dagi chiziqli akslantirishni belgilaylik:

$$E_{i,j}(e_k) = \delta_{jk} e_i,\ \mathrm{k} = 0,...,\mathrm{n}$$

bu yerda δ_{jk} - Kroneker deltasi.

Agar $n = 2m$ juft bo'lsa,

$$\mathrm{PLoc}(W_n^+) = span\{E_{i,j} : 0 \leq j \leq m-1,\ 2j+1 \leq i \leq n\}$$

Agarda $n = 2m+1$ toq bo'lsa,

$$\mathrm{PLoc}(W_n^+) = span\{E_{i,j}, E_{n,k} : 0 \leq j \leq m,\ 2j+1 \leq i \leq n, m+1 \leq k \leq n\},$$

bo'ladi.

R_n^+ ning lokal differensiallashlari.

$n \geq 5$ va $(n+1)$ – o'lchamli yechiluvchan Li algebrasi bo'lsin. $\{e_0, e_1, ..., e_n\}$ uning bazisi bo'lib, ushbu

$$[e_0, e_i] = ie_i, \qquad 1 \leq i \leq n,$$
$$[e_1, e_i] = e_{i+1}, \qquad 2 \leq i \leq n-1,$$
$$[e_2, e_i] = e_{i+2}, \qquad 3 \leq i \leq n-2$$

o'rinli bo'lsin.

[7, Teorema 2.2.1] ga ko'ra R_n^+ ning barcha differensiallashlari ichki differensiallashlardir.

Quyidagicha belgilash olamiz:

$$PLoc(R_n^+) = span\{E_{i,j} : 0 \leq j \leq 2,\ 2j+1 \leq i \leq n\}.$$

Olingan natijalar va muhokama

Teorema-1. W_n^+ ning har qanday Δ lokal differensiali ushbu

$$\Delta = ad(a) + \overline{\Delta}$$

ko‘rinishda yagona ravishda ifodalanadi. Bu yerda $a \in span\left\{e_0, e_1, ..., e_{\left[\frac{n}{2}\right]-1}\right\}$ va $\overline{\Delta} \in \mathrm{PLoc}\left(W_n^+\right)$, $[t]$-t haqiqiy sonning butun qismi. Shuningdek , $LocDerW_n^+$ fazo $\mathrm{PLoc}\left(W_n^+\right)$ idealga ega Li algebrasi bo‘ladi.

Teorema-2. *R_n^+ dagi ixtiyoriy Δ lokal differensiallash quyidagi ko‘rinishda yagona ravishda aniqlanadi:*

$$\Delta = ad(a) + \overline{\Delta}.$$

Bu yerda $a \in span\{e_0, e_1, e_2\}$ va $\overline{\Delta} \in PLoc\left(R_n^+\right)$.

Xulosa. Teorema-1. $LocDer\left(W_n^+\right)$ ning

$$\dim LocDer\left(W_n^+\right) = \begin{cases} \dfrac{n(n+4)}{4}, & n = 2m, \\ \dfrac{n^2+6n-3}{4}, & n = 2m+1. \end{cases}$$

o‘lchamli yechiluvchan Li algebrasi ekanligi ko’rsatilgan.

Teorema-2. $LocDer\left(R_n^+\right) - (3n-3)$- o‘lchamli yechiluvchan Li algebrasi ekanligi isbotlangan.

Foydalanilgan adabiyotlar ro‘yxati.

1. R.V. Kadison, Local derivations, *J. Algebra*, 130 (1990) 494-509.
2. D. R. Larson, A. R. Sourour, Local derivations and local automorphisms of *B(X)*, *Proc. Sympos. Pure Math*. 51 (1990) 187-194.
3. Ayupov Sh. A., Kudaybergenov K. K., Rakhimov I. S. 2-Local derivations on finite-dimensional Lie algebras // Linear Algebra and its Applications. 474 (2015), p. 1–11
4. Ayupov Sh. A., Kudaybergenov K. K. Local derivations on finite-dimensional Lie algebras // Linear Algebra and its Applications. 493 (2016), p. 381–398.
5. Ayupov Sh. A., Kudaybergenov K. K. Local Automorphisms on Finite-Dimensional Lie and Leibniz Algebras // Algebra, Complex Analysis and Pluripotential Theory, USUZCAMP 2017. Springer Proceedings in Mathematics and Statistics. 264 (2017), p. 31–44.
6. I.G’.Yuldashev. Local derivations on solvable Lie algebras with a filiform nilradical // “Qoraqalpog’istonda fan va ta’lim” 2021-yil 4-son (18).
7. K.K. Abdurasulov, B.A. Omirov, Maximal solvable extensions of a pure non-characteristically nilpotent Lie algebra, Preprint, arXiv:2111.07651 [math.RA].