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IKKI O'LCHAMLI UCHBURCHAKLI PANJARADA CHEKLI POTENSIALLI DISKRET SHREDINGER OPERATORI XOS QIYMATLARI

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Annotatsiya. Ishda ikki o'lchamli uchburchak panjarada chekli potensialga ega diskret Shredinger operatori tavsiflangan. Bu operatorni spektral xossalarini o'rganish uchun, dastlab operatorni ikkita invariant fazolari va undagi tasvirlari aniqlangan. So'rnga har bir operatorning potensial parametr qiymatlariga bog'liq xos qiymatlari mavjudlik shartlari o'rganilgan.

Kalit so'zlar: diskret Shredinger operatori, ikki o'lchamli uchburchak panjara, xos qiymat.

1. Kirish

Diskret Shredinger operatorlarining xos qiymatlarining mavjudlik shartlari haqida natijalar [1] adabiyotda δ-potensial uchun keltirilgan. [3]-maqolada esa δ-potensial bilan bog'liq nolokal diskret Shredinger operatorlari ko'rib chiqilgan. Ushbu mualliflar operatorning uzluksiz spektrining ikki chekkasidagi spektral qirralarning o'zgarishlarini muhokama qilganlar. Chekli o'lchamli potensialga ega bo'lgan diskret Laplas operatorining diskret xos qiymatlari sonining aniq ifodasi [5]-maqolada bayon qilingan.

K. Ando va hammualliflar [4] kvadrat, uchburchak, geksagonal, Kagome, olmos va bo'linuvchi panjaralarda aniqlangan Shredinger operatorlarini tavsiflagan hamda ushbu operatorlarning kompakt qo'llab-quvvatlovchi potensiallar bilan spektral xossalarini o'rganishgan. Ular bu turdagi Shredinger operatorlar uchun diskret spektrning chekli bo'lishi va ichki xos qiymatlarning mavjud emasligi shartlarini bergan. Uch o'lchamli uchburchakli panjarada diskret Shredinger operatorining xos qiymatlari mavjudlik shartlari potensial parametri qiymatlarga bo'liq yo'lda o'rganilgan.

Mazkur ishda, ikki o'lchamli uchburchak panjarada chekli potensialga ega diskret Shredinger operatori $L_2(\mathbb{T}^2)$ fazodagi ta'siri qaralib, bu operatorni spektral xossalarini o'rganish uchun, dastlab operatorni ikkita invariant fazolari va undagi tasvirlari aniqlangan. So'ngra har bir operatorning potensial parametr qiymatlariga bog'liq xos qiymatlari mavjudlik shartlari o'rganilgan.

2. Operator tavsifi va ishning asosiy natijalari

$L_2(\mathbb{T}^2)$ orqali $\mathbb{T}^2 = [-\pi, \pi]^2$ da aniqlangan, kvadrati bilan integrallanuvchi funksiyalar fazosini belgilaymiz. Uchburchakli panjarada diskret Shredinger operatori $L_2(\mathbb{T}^2) \to L_2(\mathbb{T}^2)$ fazoda quyidagi formula orqali aniqlanadi [4]:

$$\mathrm{H} = \mathrm{H}_0 - \mu V, \qquad (1)$$

bu yerda $\mathrm{H}_0 - \varepsilon(x)$ funksiyaga ko'paytirish operatori, ya'ni

$$(H_0 f)(x) = \varepsilon(x) f(x),$$

V esa

$$(Vf)(x) = \int_{\mathbb{T}^2} v(x-p) f(p) dp$$

ko'rinishda aniqlangan integral operator, bunda

$$\varepsilon(x) = -\frac{1}{3}(\cos x_1 + \cos x_2 + \cos(x_1 - x_2)),$$

$$v(x-p) = 1 + (\cos(x_1 - p_1) + \cos(x_2 - p_2)) + \cos\big(x_1 - x_2 - (p_1 - p_2)\big).$$

Lemma 1. (1) formula orqali aniqlangan H chiziqli, chegaralangan va o'z-o'ziga qo'shma operator bo'ladi.

Muhim spektr turg'unligi haqidagi Veyl [2] teoremasiga asosan H operatorning $\sigma_{ess}(\mathrm{H})$ muhim spektri kompakt V qo'zg'alishda o'zgarmaydi va qo'zg'almas H_0 operatorning spektri $\sigma(\mathrm{H}_0)$bilan ustma-ust tushadi. Shunday qilib, $\sigma_{ess}(\mathrm{H})$ muhim spektr $\varepsilon(s)$ funksiyaning qiymatlar to'plamidan iborat bo'ladi [6], ya'ni

$$\sigma_{ess}(\mathrm{H}) = \sigma(\mathrm{H}_0) = [m, M],\ m = min\big(\varepsilon(s)\big) = -1, \qquad M = max\big(\varepsilon(s)\big) = \frac{1}{2}$$

V operatorning musbat ekanligidan, quyidagilarni hosil qilamiz :

$$sup(Hf, f) \le sup(\mathrm{H}_0 f, f) = M(f, f).$$

Bu yerdan va $sup\ \sigma(H) = \sup\limits_{\|f\|=1}(Hf, f)$ tenglikdan H operator, muhim spektrdan o'ngda xos qiymatga ega bo'la olmaydi.

Lemma 2. (1) formula oraqali aniqlangan operator $L_2^e(\mathbb{T}^2)$ va $L_2^o(\mathbb{T}^2)$ funksiyalar fazolariga nisbatan invariantdir.

H operator $L_2^e(\mathbb{T}^2)$ va $L_2^o(\mathbb{T}^2)$ funksiyalar fazolarida mos ravishda quyidagi ko'rinishda:

$$H_e = H_0 - \mu V^e, \qquad H_o = H_0 - \mu V^o,$$

bu yerda

$$(V^e f)(x) = -\int\limits_{\mathbb{T}^2} f(p)dp - \cos x_1 \int\limits_{\mathbb{T}^2} \cos p_1 f(p)dp - \cos x_2 \int\limits_{\mathbb{T}^2} \cos p_2 f(p)dp$$

$$- cos(x_1 - x_2) \int\limits_{\mathbb{T}^2} cos(p_1 - p_2) f(p)dp,$$

$$(V^o f)(x) = -\sin x_1 \int\limits_{\mathbb{T}^2} \sin p_1 f(p)dp - \sin x_2 \int\limits_{\mathbb{T}^2} \sin p_2 f(p)dp$$

$$- \sin(x_1 - x_2) \int\limits_{\mathbb{T}^2} \sin(p_1 - p_2) f(p)dp.$$

1-teorema. Ixtiyoriy $z < -1$ uchun shunday $\mu > 0$ topilib, z soni H_o operatorning xos qiymati bo'ladi.

2-teorema. Ixtiyoriy $z < -1$ uchun shunday $\mu > 0$ topilib, z soni H_e operatorning xos qiymati bo'ladi.

3. Asosiy natijalar isboti.

Quyidagi belgilashlarni kiritamiz:

$$\Delta^o(z) = \begin{vmatrix} \mu A - 1 & \mu B & \mu C \\ \mu B & \mu A - 1 & -\mu C \\ \mu C & -\mu C & \mu D - 1 \end{vmatrix}, \qquad z < -1$$

$$A = \int\limits_{\mathbb{T}^2} \frac{sin^2 p_1 dp_1 dp_2}{\varepsilon(p_1, p_2) - z} = \int\limits_{\mathbb{T}^2} \frac{sin^2 p_2 dp_1 dp_2}{\varepsilon(p_1, p_2) - z},$$

$$B = \int\limits_{\mathbb{T}^2} \frac{\sin p_1 \sin p_2\, dp_1 dp_2}{\varepsilon(p_1, p_2) - z},$$

$$C = \int\limits_{\mathbb{T}^2} \frac{\sin p_1 \sin(p_1 - p_2)\, dp_1 dp_2}{\varepsilon(p_1, p_2) - z} = -\int\limits_{\mathbb{T}^2} \frac{\sin p_2 \sin(p_1 - p_2)\, dp_1 dp_2}{\varepsilon(p_1, p_2) - z},$$

$$D = \int_{\mathbb{T}^2} \frac{sin^2(p_1 - p_2)dp_1dp_2}{\varepsilon(p_1,p_2) - z}.$$

1-teoremani isbotlash uchun quyidagi lemmadan foydalanamiz:

3-lemma. $z < -1$ soni H operatorning xos qiymati bo'lishi uchun $\Delta(z) = 0$ bo'lishi zarur va yetarli.

3-lemma isboti: Faraz qilaylik z xos qiymat bo'lsin, ya'ni

$$(Hf)(x_1,x_2) = zf(x_1,x_2) \qquad (2)$$

Quyidagi belgilashlarni kiritamiz:

$$a = \int_{\mathbb{T}^2} sin\, p_1\, f(p_1,p_2)d(p_1,p_2),$$

$$b = \int_{\mathbb{T}^2} \sin p_2\, f(p_1,p_2)d(p_1,p_2),$$

$$c = \int_{\mathbb{T}^2} \sin(p_1 - p_2)\, f(p_1,p_2)d(p_1,p_2),$$

U holda (2) tenglama ushbu ko'rinishni oladi:

$$\varepsilon(x_1,x_2)f(x_1,x_2) - \mu a \sin x_1 - \mu b \sin x_2 - \mu c \sin(x_1 - x_2) = zf(x_1,x_2)$$

$z \notin \left[-1,\frac{1}{2}\right]$ bo'lganligidan $\varepsilon(x_1,x_2) - z \neq 0$ kelib chiqadi. U holda

$$f(x_1,x_2) = \frac{\mu a \sin x_1 + \mu b \sin x_2 + \mu c \sin(x_1 - x_2)}{\varepsilon(x_1,x_2) - z} \qquad (3)$$

tenglikga ega bo'lamiz. (3) ni a, b, c ifodalarni integral qiymatiga qo'yamiz

$$a = \int_{\mathbb{T}^2} sin\, p_1 \frac{\mu a \sin p_1 + \mu b \sin p_2 + \mu c \sin(p_1 - p_2)}{\varepsilon(p_1,p_2) - z} d(p_1,p_2)$$

$$b = \int_{\mathbb{T}^2} \sin p_2 \frac{\mu a \sin p_1 + \mu b \sin p_2 + \mu c \sin(p_1 - p_2)}{\varepsilon(p_1,p_2) - z} d(p_1,p_2)$$

$$c = \int_{\mathbb{T}^2} \sin(p_1 - p_2) \frac{\mu_2 a \sin p_1 + \mu_2 b \sin p_2 + \mu_3 c \sin(p_1 - p_2)}{\varepsilon(p_1,p_2) - z} d(p_1,p_2)$$

Bundan quyidagi tenglamalar sistemasiga kelamiz:

$$\left(-1 + \mu \int_{\mathbb{T}^2} \frac{sin^2 p_1 dp_1 dp_2}{\varepsilon(p_1,p_2) - z}\right) a + \mu \int_{\mathbb{T}^2} \frac{sin\, p_1\, sin\, p_2\, dp_1 dp_2}{\varepsilon(p_1,p_2) - z} b + \mu \int_{\mathbb{T}^2} \frac{sin\, p_1\, sin(p_1 - p_2)\, dp_1 dp_2}{\varepsilon(p_1,p_2) - z} c = 0$$

$$\mu \int_{\mathbb{T}^2} \frac{sin\, p_1\, sin\, p_2\, dp_1 dp_2}{\varepsilon(p_1,p_2) - z} a + \left(-1 + \mu \int_{\mathbb{T}^2} \frac{sin^2 p_2 dp_1 dp_2}{\varepsilon(p_1,p_2) - z}\right) b + \mu \int_{\mathbb{T}^2} \frac{sin\, p_2\, sin(p_1 - p_2)\, dp_1 dp_2}{\varepsilon(p_1,p_2) - z} c = 0$$

$$\mu \int_{\mathbb{T}^2} \frac{\sin p_1 \sin(p_1 - p_2)\, dp_1 dp_2}{\varepsilon(p_1,p_2) - z} a + \mu \int_{\mathbb{T}^2} \frac{\sin p_2 \sin(p_1 - p_2)\, dp_1 dp_2}{\varepsilon(p_1,p_2) - z} b + \left(-1 + \mu \int_{\mathbb{T}^2} \frac{sin^2(p_1 - p_2) dp_1 dp_2}{\varepsilon(p_1,p_2) - z}\right) c = 0 \qquad (4)$$

Yuqoridagi belgilashlardan (4) tenglamalar sistemasi quyidagi ko'rinishga keladi:

$$\begin{cases}(-1+\mu A)a+\mu Bb+\mu Cc=0\\ \mu Ba+(\mu A-1)b-\mu Cc=0\\ \mu Ca-\mu Cb+(\mu D-1)c=0\end{cases} \quad (5)$$

Bu 3 noma'lumli va 3 ta tenglamalar sistemasiga mos $\Delta^o(z)$ determinantni tuzamiz:

$$\Delta^o(z)=\begin{vmatrix}\mu A-1 & \mu B & \mu C\\ \mu B & \mu A-1 & -\mu C\\ \mu C & -\mu C & \mu D-1\end{vmatrix}$$

(3) ga ko'ra $(a,b,c)\neq(0,0,0)$, (5) sistema noldan farqli yechimga ega bo'lishi uchun $\Delta^o(z)=0$ bo'lishi zarur.

Yetarliligi. $\Delta^o(z)=0$ bo'lsin, u holda (5) sistema $(a,b,c)\neq(0,0,0)$ yechimga ega bo'ladi. Quyidagicha funksiyani qaraymiz

$$g(x_1,x_2)=\frac{\mu a\sin x_1+\mu b\sin x_2+\mu c\sin(x_1-x_2)}{\varepsilon(x_1,x_2)-z} \quad (6)$$

Tekshirib ko'rish mumkunki, $(Hg)(x_1,x_2)=zg(x_1,x_2)$ bo'ladi. Demak, z H operatorning xos qiymati, (6) esa H operatorning xos funksiyasi bo'ladi. 3-lemma isbotlandi.

1-teorema isboti : (5) ga ko'ra $\Delta^o(z)$ quyidagi ko'rinishda bo'ladi:

$$\Delta^o(z)=(\mu D-1)[(\mu A-1)^2-(\mu B)^2]-2(\mu C)^2(\mu B+\mu A-1).$$

Tenglikning o'ng tomonini ko'paytuvchilarga ajratamiz:

$$\Delta^o(z)=(\mu A+\mu B-1)\ [(\mu D-1)(\mu A-1-\mu B)-2(\mu C)^2].$$

$\Delta^o(z)$=0 bo'lishi uchun $\mu A+\mu B-1$=0 ekanligini ko'rsatamiz:

$$\mu(A+B)=1$$

$$A=\int_{\mathbb{T}^2}\frac{\sin^2 p_1dp_1dp_2}{\varepsilon(p_1,p_2)-z}=\int_{\mathbb{T}^2}\frac{\sin^2 p_2dp_1dp_2}{\varepsilon(p_1,p_2)-z}>0,$$

$$B=\int_{\mathbb{T}^2}\frac{\sin p_1\sin p_2dp_1dp_2}{\varepsilon(p_1,p_2)-z}.$$

Koshi-Bunyakovskiy formulasiga ko'ra:

$$B=\int_{\mathbb{T}^2}\frac{\sin p_1\sin p_2dp_1dp_2}{\varepsilon(p_1,p_2)-z}\le\sqrt{\int_{\mathbb{T}^2}\frac{\sin^2 p_1dp_1dp_2}{\varepsilon(p_1,p_2)-z}}\sqrt{\int_{\mathbb{T}^2}\frac{\sin^2 p_2dp_1dp_2}{\varepsilon(p_1,p_2)-z}}=\int_{\mathbb{T}^2}\frac{\sin^2 p_1dp_1dp_2}{\varepsilon(p_1,p_2)-z}=A,$$

Demak, $A+B>0$, $\mu=\frac{1}{A+B}>0$ bo'ladi.

Agar $\mu=\frac{1}{A+B}$ deb olsak, u holda $\Delta^o(z)$=0 bo'ladi. Bu esa 3-lemmaga ko'ra $z<-1$ soni H operatorning xos qiymati bo'lishini anglatadi. 1-teorema isbotlandi.

Quyidagi belgilashlarni kiritamiz:

$$\Delta^o(z)=\begin{vmatrix}\mu A-1 & \mu B & \mu B & \mu C\\ \mu B & \mu D-1 & \mu E & \mu F\\ \mu B & \mu E & \mu D-1 & \mu F\\ \mu C & \mu F & \mu F & \mu G-1\end{vmatrix},$$

$$A=\int\limits_{\mathbb{T}^2}\frac{dp_1dp_2}{\varepsilon(p_1,p_2)-z},$$

$$B=\int\limits_{\mathbb{T}^2}\frac{\cos p_1\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}=\int\limits_{\mathbb{T}^2}\frac{\cos p_2\,dp_1dp_2}{\varepsilon(p_1,p_2)-z},$$

$$C=\int\limits_{\mathbb{T}^2}\frac{\cos(p_1-p_2)\,dp_1dp_2}{\varepsilon(p_1,p_2)-z},$$

$$D=\int\limits_{\mathbb{T}^2}\frac{\cos^2 p_1dp_1dp_2}{\varepsilon(p_1,p_2)-z}=\int\limits_{\mathbb{T}^2}\frac{\cos^2 p_2dp_1dp_2}{\varepsilon(p_1,p_2)-z},$$

$$E = \int_{\mathbb{T}^2} \frac{\cos p_1 \cos p_2 \, dp_1 dp_2}{\varepsilon(p_1, p_2) - z},$$

$$F = \int_{\mathbb{T}^2} \frac{\cos p_1 \cos(p_1 - p_2) \, dp_1 dp_2}{\varepsilon(p_1, p_2) - z} = \int_{\mathbb{T}^2} \frac{\cos p_2 \cos(p_1 - p_2) \, dp_1 dp_2}{\varepsilon(p_1, p_2) - z},$$

$$G = \int_{\mathbb{T}^2} \frac{\cos^2(p_1 - p_2) dp_1 dp_2}{\varepsilon(p_1, p_2) - z}.$$

2-teoremani isbotlash uchun quyidagi lemmadan foydalanamiz:

4-lemma. $z < -1$ soni H operatorning xos qiymati bo'lishi uchun $\Delta^e(z) = 0$ bo'lishi zarur va yetarli.

4-lemma isboti: Faraz qilaylik z xos qiymat bo'lsin, ya'ni

$$(Hf)(x_1, x_2) = zf(x_1, x_2) \qquad (7)$$

Quyidagi belgilashlarni kiritamiz:

$$a = \int_{\mathbb{T}^2} f(p_1, p_2) d(p_1, p_2)$$

$$b = \int_{\mathbb{T}^2} \cos p_1 \, f(p_1, p_2) d(p_1, p_2)$$

$$c = \int_{\mathbb{T}^2} \cos p_2 \, f(p_1, p_2) d(p_1, p_2)$$

$$d = \int_{\mathbb{T}^2} \cos(p_1 - p_2) \, f(p_1, p_2) d(p_1, p_2)$$

u holda (7) tenglama ushbu ko'rinishni oladi:

$$\varepsilon(x_1, x_2) f(x_1, x_2) - \mu a - \mu b \cos x_1 - \mu c \cos x_2 - \mu d \cos(x_1 - x_2) = zf(x_1, x_2)$$

$z \notin \left[-1, \frac{1}{2}\right]$ bo'lganligidan $\varepsilon(x_1, x_2) - z \neq 0$ kelib chiqadi. U holda

$$f(x_1, x_2) = \frac{\mu a + \mu b \cos x_1 + \mu c \cos x_2 + \mu d \cos(x_1 - x_2)}{\varepsilon(x_1, x_2) - z} \qquad (8)$$

tenglikga ega bo'lamiz. (3) ni a, b, c, d ifodalarni integral qiymatiga qo'yamiz

$$a = \int_{\mathbb{T}^2} \frac{\mu a + \mu b \cos x_1 + \mu c \cos x_2 + \mu d \cos(x_1 - x_2)}{\varepsilon(x_1, x_2) - z} d(p_1, p_2)$$

$$b = \int_{\mathbb{T}^2} \cos p_1 \frac{\mu a + \mu b \cos x_1 + \mu c \cos x_2 + \mu d \cos(x_1 - x_2)}{\varepsilon(x_1, x_2) - z} d(p_1, p_2)$$

$$c = \int_{\mathbb{T}^2} \cos p_2 \frac{\mu a + \mu b \cos x_1 + \mu c \cos x_2 + \mu d \cos(x_1 - x_2)}{\varepsilon(x_1, x_2) - z} d(p_1, p_2)$$

$$d = \int_{\mathbb{T}^2} \cos(p_1 - p_2) \frac{\mu a + \mu b \cos x_1 + \mu c \cos x_2 + \mu d \cos(x_1 - x_2)}{\varepsilon(x_1, x_2) - z} d(p_1, p_2)$$

Bundan quyidagi tenglamalar sistemasiga kelamiz:

$$\left(-1+\mu\int\limits_{\mathbb{T}^2}\frac{dp_1dp_2}{\varepsilon(p_1,p_2)-z}\right)a+\mu\int\limits_{\mathbb{T}^2}\frac{\cos p_1\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}b+\mu\int\limits_{\mathbb{T}^2}\frac{\cos p_2\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}c$$
$$+\mu\int\limits_{\mathbb{T}^2}\frac{cos(p_1-p_2)\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}d=0$$
$$\mu\int\limits_{\mathbb{T}^2}\frac{\cos p_1\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}a+\left(-1+\mu\int\limits_{\mathbb{T}^2}\frac{\cos^2 p_1dp_1dp_2}{\varepsilon(p_1,p_2)-z}\right)b+\mu\int\limits_{\mathbb{T}^2}\frac{\cos p_1\cos p_2\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}c$$
$$+\mu\int\limits_{\mathbb{T}^2}\frac{\cos p_1\cos(p_1-p_2)\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}d=0$$
$$\mu\int\limits_{\mathbb{T}^2}\frac{\cos p_2\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}a+\mu\int\limits_{\mathbb{T}^2}\frac{\cos p_1\cos p_2\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}b+\left(-1+\mu\int\limits_{\mathbb{T}^2}\frac{\cos^2 p_2dp_1dp_2}{\varepsilon(p_1,p_2)-z}\right)c$$
$$+\mu\int\limits_{\mathbb{T}^2}\frac{\cos p_2\cos(p_1-p_2)\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}d=0$$
$$\mu\int\limits_{\mathbb{T}^2}\frac{cos(p_1-p_2)\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}a+\mu\int\limits_{\mathbb{T}^2}\frac{\cos p_1\cos(p_1-p_2)\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}b+\mu\int\limits_{\mathbb{T}^2}\frac{\cos p_2\cos(p_1-p_2)\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}c$$
$$+\left(-1+\mu\int\limits_{\mathbb{T}^2}\frac{\cos^2(p_1-p_2)dp_1dp_2}{\varepsilon(p_1,p_2)-z}\right)d=0 \qquad (9)$$

Yuqoridagi belgilashlardan (9) tenglamalar sistemasi quyidagi ko'rinishga keladi:

$$\begin{cases}(\mu A-1)a+\mu Bb+\mu Bc+\mu Cd=0\\ \mu Ba+(\mu D-1)b+\mu Ec+\mu Fd=0\\ \mu Ba+\mu Eb+(\mu D-1)c+\mu Fd=0\\ \mu Ca+\mu Fb+\mu Fc+(\mu G-1)d=0\end{cases} \qquad (10)$$

Bu 4 noma'lumli tenglamalar sistemasiga mos $\Delta^e(z)$ determinantni tuzamiz:

$$\Delta^e(z)=\begin{vmatrix}\mu A-1 & \mu B & \mu B & \mu C\\ \mu B & \mu D-1 & \mu E & \mu F\\ \mu B & \mu E & \mu D-1 & \mu F\\ \mu C & \mu F & \mu F & \mu G-1\end{vmatrix}$$

(3) ga ko'ra $(a,b,c,d)\neq(0,0,0,0)$, (9) sistema noldan farqli yechimga ega bo'lishi uchun $\Delta^o(z)=0$ bo'lishi zarur.

Yetarliligi. $\Delta^o(z)=0$ bo'lsin, u holda (9) sistema $(a,b,c,d)\neq(0,0,0,0)$ yechimga ega bo'ladi. Quyidagicha funksiyani qaraymiz

$$g(x_1,x_2)=\frac{\mu a+\mu b\cos x_1+\mu c\cos x_2+\mu d\cos(x_1-x_2)}{\varepsilon(x_1,x_2)-z} \qquad (11)$$

Tekshirib ko'rish mumkunki, $(Hg)(x_1,x_2)=zg(x_1,x_2)$ bo'ladi. Demak, z H operatorning xos qiymati, (11) esa H operatorning xos funksiyasi bo'ladi. Lemma isbotlandi.

Teoremani isbotlash uchun z ning xos qiymat bo'lishi uchun zarur va yetarli shartlar ko'rib chiqiladi. Bu shartlar determinant sistemaning yechimlari va integral ifodalar orqali ifodalanadi. Olingan xos funksiyalar shakli aniq ifodalanadi va ular operator tenglamasini qanoatlantirishi ko'rsatiladi.

Natijalar shuni ko‘rsatadiki, agar determinant sharti bajarilsa, H operatorida asosiy spektrdan tashqari xos qiymatlar bo‘lishi mumkin. Bu qiymatlar potensial funksiyalar va integral yadroning tuzilishiga bog‘liq bo‘ladi.

2-teorema isboti : (9) dan $\Delta^e(z)$ quyidagi ko’rinishda bo’ladi

$$\Delta^e(z)=\begin{vmatrix} \mu A-1 & \mu B & \mu B & \mu C \\ \mu B & \mu D-1 & \mu E & \mu F \\ \mu B & \mu E & \mu D-1 & \mu F \\ \mu C & \mu F & \mu F & \mu G-1 \end{vmatrix}$$

Hisoblashlarni bajarganimizdan so’ng tenglikning o’ng tomonidan quyidagi umumiy ko’paytuvchi qavs oldiga chiqadi:

$$\Delta^e(z)=(\mu E-\mu D+1)\big((\mu A-1)[2(\mu F)^2-(\mu G-1)(\mu E+\mu D-1)] + 2\mu B[\mu B(\mu G-1)-\mu C\mu F]-\mu C[2\mu B\mu F-\mu C(\mu E+\mu D-1)]\big).$$

$\Delta^e(z)$=0 bo’lishi uchun $\mu E-\mu D+1=0$ ekanligini ko’rsatamiz. $\mu(D-E)=1$

$$D=\int_{\mathbb{T}^2}\frac{cos^2p_1dp_1dp_2}{\varepsilon(p_1,p_2)-z}=\int_{\mathbb{T}^2}\frac{cos^2p_2dp_1dp_2}{\varepsilon(p_1,p_2)-z}>0,$$

$$E=\int_{\mathbb{T}^2}\frac{cos\,p_1\,cos\,p_2\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}.$$

Koshi-Bunyakovskiy formulasiga ko’ra:

$$E=\int_{\mathbb{T}^2}\frac{cos\,p_1\,cos\,p_2\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}\le\sqrt{\int_{\mathbb{T}^2}\frac{cos^2\,p_1\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}}\sqrt{\int_{\mathbb{T}^2}\frac{cos^2\,p_2\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}}=\int_{\mathbb{T}^2}\frac{cos^2\,p_1\,dp_1dp_2}{\varepsilon(p_1,p_2)-z}=D.$$

Demak, $D-E>0$, $\mu=\frac{1}{D-E}>0$ bo’ladi.

Agar $\mu=\frac{1}{D-E}$ deb olsak, u holda $\Delta^e(z)=0$ bo’ladi. Bu esa 4-lemmaga ko’ra $z<-1$ soni H operatorning xos qiymati bo’lishini anglatadi. 2-teorema isbotlandi.

Foydalanilgan adabiyotlar

[1]. F. Hiroshima, I. Sasaki, T. Shirai, and A. Suzuki, “Note on the spectrum of discrete Schrödinger operators,” arxiv: 1209.0522 (2012).
[2]. M. Reed, B. Simon, Methods of Modern Mathematical Physics, Vol. 4, Analysis of Operators, Academic Press, London, 1980.
[3]. F. Hiroshima, Fumio, and L. Joozsef, “The spectrum of non-local discrete Schrödinger operators with a δ potential,” Pacif. J. Math. Ind. 6,1–6 (2014).
[4]. K. Ando, I. Hiroshi, H. Hisashi, “Spectral properties of Schrödinger operators on perturbed lattices”, Ann. Henri Poincaré, 17, 2103 (2016).
[5]. Y. Hayashi, Y. Higuchi, Y. Nomura, and O. Ogurisu, “On the number of discrete eigenvalues of a discrete Schördinger operator with a finitely supported potential,” Lett. Math. Phys. 106, 1465–1478 (2016).
[6]. M. Muminov, J. Pardaev, “The Spectrum of Discrete Schrödinger Operator on a Three Dimensional Triangular Lattice with a Finite-range Potential”, Lobachevskii Journal of Mathematics, Vol. 45, No. 4, pp. 1722–1728, 2024.